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DETONATION WAVE PROPAGATION IN ROTATIONAL GAS FLOWS

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This paper studies the propagation of detonation and shock waves in vortex gas flows, in which the initial pressure, density, and velocity are generally functions of the coordinate — the distance from the symmetry axis. Rotational axisymmetric flow having a transverse velocity component in addition to a nonuniform longitudinal velocity is considered. The possibility of propagation of Chapman–Jouguet detonation waves in rotating flows is analyzed. A necessary conditions for the existence of a Chapman–Jouguet wave is obtained.

Key words: *vortex, shock wave, detonation wave, axisymmetric flow, discontinuity surface, Chapman–Jouguet wave.*

Vorticity Change in Rotational Flows on a Discontinuity Surface. We consider an axisymmetric rotational flow characterized by a transverse velocity component in addition to a nonuniform longitudinal velocity. For an ideal perfect gas, such a flow is described by the following system of equations:

$$\frac{\partial\rho}{\partial t} + v\,\frac{\partial\rho}{\partial r} + \rho\,\frac{\partial v}{\partial r} + \frac{\rho v}{r} = 0,$$

$$\rho\Big(\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial r} - \frac{w^2}{r}\Big) + \frac{\partial p}{\partial r} = 0,$$

$$\frac{\partial w}{\partial t} + v\,\frac{\partial w}{\partial r} + \frac{vw}{r} = 0, \qquad \frac{\partial u}{\partial t} + v\,\frac{\partial u}{\partial r} = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\Big(\frac{p}{\rho^\gamma}\Big) + v\,\frac{\partial}{\partial r}\Big(\frac{p}{\rho^\gamma}\Big) = 0.$$

Here u, v, and w are the corresponding velocity components in the cylindrical coordinates (x, r, φ), ρ is the density, p is the pressure, and t is time.

The steady-state solution of system (1) is written as

$$u = u^0(r), \qquad v^0 = 0, \qquad w = w^0(r), \qquad \rho = \rho^0(r), \qquad p^0 = \int \frac{\rho^0 w^{02}}{r}\,dr. \tag{2}$$

In this case, the vortex vector $2\boldsymbol{\omega} = \operatorname{rot}\boldsymbol{V}$ has the components

$$\omega_{0r} = 0, \qquad \omega_{0x} = \frac{1}{2r}\,\frac{\partial r w^0}{\partial r}, \qquad \omega_{0\varphi} = -\frac{1}{2}\,\frac{\partial u^0}{\partial r}.$$

If, at the initial time, an explosion occurs on the symmetry axis, resulting in the formation of an explosive shock wave, or the mixture is ignited to form a detonation wave, then a cylindrical shock or detonation wave (DW) will propagate in the flow.

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We seek an expression for the vortex vector components immediately behind the discontinuity surface. For this, we determine the quantities $\partial u/\partial r$ and $\partial w/\partial r$ immediately behind the jump at $r = R(t)$ [$R(t)$ is the law of motion for the discontinuity surface]. Using Eqs. (1), we obtain expressions for these derivatives in terms of postshock quantities and their derivatives with respect to time:

$$\frac{\partial u}{\partial r}\Big|_s = -\frac{\dot u_1}{v_1 - \dot R}, \qquad \frac{\partial w}{\partial r}\Big|_s = -\frac{\dot w_1 + v_1 w_1/R}{v_1 - \dot R}.$$

In these formulas, the dot denotes differentiation of the corresponding quantities on the discontinuity surface with respect to time; the subscript 1 refers to the postshock parameters.

For the vortex vector components behind the discontinuity, we obtain

$$\omega_{1\varphi} = \frac{1}{2}\,\frac{\dot u_1}{\dot v_1 - \dot R}, \qquad \omega_{1x} = \frac{1}{2}\Big(\frac{w_1}{R} - \frac{\dot w_1 + v_1 w_1/R}{v_1 - R}\Big), \qquad \omega_{1r} = 0.$$

For the vortex vector components ahead of the shock, we have

$$\omega_{0\varphi} = -\frac{1}{2}\,\frac{\dot u^0}{\dot R}, \qquad \omega_{0x} = \frac{1}{2}\Big(\frac{w^0}{R} + \frac{\dot w^0}{\dot R}\Big).$$

On the discontinuity surface, the tangential velocity components are continuous, i.e., $u^0 = u_1$ and $w^0 = w_1$. Taking into account this circumstance and the law of conservation of mass at the shock, we obtain

$$\frac{\omega_{1x}}{\omega_{0x}} = \frac{\omega_{1\varphi}}{\omega_{0\varphi}} = \frac{D}{D - v_1} = \frac{\rho_1}{\rho^0}. \qquad (3)$$

The rate of propagation of the discontinuity $\dot R = D$.

From (3) it immediately follows that the quantities ω_x/ρ and ω_φ/ρ are continuous in transition through the discontinuity surface irrespective of whether the discontinuity is a shock wave or a detonation wave.

Thus, for the class of flows described, the law of conservation of the quantity $\boldsymbol{\omega}/\rho$ is satisfied on the discontinuity surface although the quantities $\boldsymbol{\omega}$ and ρ undergo a discontinuity. We note that in transition through discontinuities, the flow vorticity increases in proportion to the density ratio. Therefore, for the same shock velocity, the vorticity behind a shock wave is higher than that behind a DW.

This conclusion is also valid for plane shear flows.

Possibility of Propagation of Chapman–Jouguet DWs in Rotational Flows. We consider the propagation of a divergent detonation wave in rotational gas flows with the initial parameter distribution (2). The detonation wave is treated as a discontinuity surface on which combustion of a unit mass of gas leads to release of heat Q, whose amount depends on the coordinate $Q = Q(r)$. The flow behind the detonation front is described by the Euler equations (1).

At the front of a Chapman–Jouguet DW, the following relations are satisfied [1]:

$$\rho_J = \rho^0\,\frac{\gamma+1}{\gamma+q_J}, \qquad p_J = p^0\,\frac{\gamma+q_J}{(\gamma+1)q_J}, \qquad v_J = D_J\,\frac{1-q_J}{\gamma+1},$$

$$u_J = u^0, \qquad w_J = w^0, \qquad D_J^2 = \frac{a^{02}}{q_J}, \qquad a^{02} = \gamma\,\frac{p^0}{\rho^0}, \qquad a_J^2 = \frac{\gamma p_J}{\rho_J}, \qquad (4)$$

$$q_J^2 - 2q_J[1 + (\gamma^2-1)Q/a^{02}] + 1 = 0.$$

Here the subscript J denotes the gas parameters and the Chapman–Jouguet wave velocity. The above system of gas-dynamic equations, being hyperbolic, has three sets of characteristics, for which the corresponding characteristic relations are satisfied [1]. In this case, if on a certain fairly smooth curve $r_0(t)$, the values of the functions satisfy one of the characteristic relations but do not satisfy another characteristic relation, this curve is the envelope of the corresponding set of characteristics of Eqs. (1) and the solution in its neighborhood should be sought in the form

$$p(r,t) = p_0(t) + p_1(t)\sqrt{r_0(t)-r} + p_2(t)(r_0(t)-r) + p_3(t)(r_0(t)-r)^{3/2} + \ldots \qquad (5)$$

(similarly for the remaining sought parameters) [1].

This approach was used to determine the conditions of existence of plane Chapman–Jouguet detonation waves in external electric and magnetic fields [2] and to analyze the DW propagation in nonuniform media [3]. For arbitrary systems of quasilinear first-order partial equations, the conditions of existence were explored and the form of the asymptotic expansion of the solution was determined in the neighborhood of the envelope of characteristic surfaces on which initial function values [4] are specified. Convergence of the corresponding series is proved in [5].

Substituting expansions (5) into Eqs. (1), we obtain an infinite system of algebraic equations for the expansion coefficients. The equations for the coefficients with subscripts 0, 1, and 2 have the form

$$\begin{gathered}\rho_1(D-v_0)-\rho_0 v_1=0, \qquad p_1-\rho_0(D-v_0)v_1=0, \qquad \rho_0 p_1-\gamma p_0\rho_1=0,\\ u_1(D-v_0)=0, \qquad w_1(D-v_0)=0;\end{gathered} \tag{6}$$

$$\begin{gathered}\rho_2(D-v_0)-\rho_0 v_2=\rho_1 v_1-\dot\rho_0-\rho_0 v_0/r_0,\\ \rho_0 v_2(D-v_0)-p_2=\rho_0 w_0^2/r_0-\rho_0\dot v_0,\\ (D-v_0)(\rho_0 p_2-\gamma p_0\rho_2)=\gamma p_0\dot\rho_0-\rho_0\dot p_0+(\gamma-1)\rho_0 p_1 v_1/2,\\ u_2(D-v_0)=-\dot u_0, \qquad w_2(D-v_0)=-\dot w_0-v_0 w_0/r_0.\end{gathered} \tag{7}$$

Here differentiation with respect to t is denoted by a dot and $D=\dot r_0$. Since the DW propagates in the Chapman–Jouguet mode, $D-v_0=a_0$, i.e., the characteristic relation is satisfied [1]. From this it follows that the determinants of systems (6) and (7), and the systems of all subsequent approximations for the expansion coefficients v_k, ρ_k, and p_k are equal to zero.

The expansion coefficients u_k and w_k are immediately found from known values of the previous coefficients, and $u_1=w_1=0$. Therefore, the series expansion for the velocities u and w has the form $u=u_0+u_2(\dot r_0-r)+u_3(\dot r_0-r)^{3/2}+\ldots$.

For consistency of the linear system of equations (7) and all subsequent systems, it is necessary that the extended determinant of the system be equal to zero. This condition implies the relation

$$\frac{\gamma-1}{2}\rho_0 p_1 v_1-\rho_0\dot p_0+\gamma p_0\rho_1 v_1+\rho_0^2(D-v_0)\Big(\frac{w_0^2}{r_0}-\dot v_0\Big)-\frac{\gamma p_0\rho_0 v_0}{r_0}=0,$$

from which, using (6), we have

$$\rho_1^2=\frac{2\rho_0^3}{\gamma(\gamma+1)p_0}\Big[\dot v_0-\frac{w_0^2}{r_0}+(D-v_0)\Big(\frac{v_0}{r_0}+\frac{\dot p_0}{\gamma p_0}\Big)\Big].$$

Similar expressions can be obtained for the next expansion coefficients v_k, p_k, and ρ_k. Taking into account the consistency relations, we can completely construct series of the indicated form and thus to determine the solution of Eq. (1) in a certain neighborhood of the curve $r=r_0(t)$, which, in our case, is the neighborhood of the DW propagating in the Chapman–Jouguet mode.

The desired solution exists if the following condition is satisfied:

$$\dot v_0-\frac{w_0^2}{r_0}+a_0\Big(\frac{v_0}{r_0}+\frac{\dot p_0}{\gamma p_0}\Big)\geqslant 0; \tag{8}$$

the equality sign determines satisfaction of the corresponding relation along the characteristic.

Inequality (8), together with the expressions for the parameters behind a Chapman–Jouguet detonation wave (4), defines the necessary condition for the existence of a Chapman–Jouguet wave ($p_0=p_J$, $v_0=v_J$, $a_0=a_J$, $\rho_0=\rho_J$, and $D=D_J$) which propagates in a medium with the distribution of the rotational flow parameters (2) and a variable law of heat release $Q=Q(r)$. Thus, the necessary condition for the existence of Chapman–Jouguet waves has the form

$$\dot v_J-\frac{w_0^2}{r_J}+a_J\Big(\frac{v_J}{r_J}+\frac{\dot p_J}{\gamma p_J}\Big)\geqslant 0.$$

Let us consider the case where $Q=\text{const}$ and $q_J\ll 1$. Then, $D_J^2=2(\gamma^2-1)Q$, $v_J=D_J/(\gamma+1)$, $p_J=\rho^0 D_J^2/(\gamma+1)$, and $\rho_J=(\gamma+1)\rho^0/\gamma$.

In this case, the necessary condition for the existence of Chapman–Jouguet waves has the form

$$\frac{1}{\gamma+1}\frac{\partial\ln\rho^0}{\partial\ln r}+\frac{\gamma}{(\gamma+1)^2}-\frac{w_0^2}{D_J^2}\geqslant 0.$$

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